

Capas

Para $n \in \mathbb{N}$, sean los conjuntos:

$$F_n = \{(x_0, x_1, \dots, x_{n-1}) \in \{0,1\}^n \mid x_0 \geq x_1 \geq \dots \geq x_{n-1}\}$$

$$G_n = \{(x_0, x_1, \dots, x_{n-1}) \in \{0,1\}^n \mid x_0 + x_1 + \dots + x_{n-1} = 1\}$$

Sea la función $H_1: F_1 \rightarrow G_2$ definida así:

$$0 \mapsto (0,1) = (f_{1,0}(0), f_{1,1}(0))$$

$$1 \mapsto (1,0) = (f_{1,0}(1), f_{1,1}(1))$$

$$H_1(x_0) = (f_{1,0}(x_0), f_{1,1}(x_0))$$

donde,

$$f_{1,0}(x_0) = x_0$$

$$f_{1,1}(x_0) = 1 - x_0 = 1 - f_{1,0}(x_0)$$

Sea la función $H_2: F_2 \rightarrow G_3$ definida así:

$$(0,0) \mapsto (0,0,1) = (f_{2,0}(0,0), f_{2,1}(0,0), f_{2,2}(0,0))$$

$$(1,0) \mapsto (0,1,0) = (f_{2,0}(1,0), f_{2,1}(1,0), f_{2,2}(1,0))$$

$$(1,1) \mapsto (1,0,0) = (f_{2,0}(1,1), f_{2,1}(1,1), f_{2,2}(1,1))$$

$$H_2(x_0, x_1) = (f_{2,0}(x_0, x_1), f_{2,1}(x_0, x_1), f_{2,2}(x_0, x_1))$$

donde,

$$f_{2,0}(x_0, x_1) = x_1 = f_{1,0}(x_1)$$

$$f_{2,1}(x_0, x_1) = (1 - x_1)x_0 = (1 - f_{2,0}(x_0, x_1))f_{1,0}(x_0)$$

$$f_{2,2}(x_0, x_1) = (1 - x_1)(1 - x_0) = (1 - f_{2,0}(x_0, x_1))f_{1,1}(x_0)$$

Sea la función $H_3: F_3 \rightarrow G_4$ definida así:

$$(0,0,0) \mapsto (0,0,0,1) = (f_{3,0}(0,0,0), f_{3,1}(0,0,0), f_{3,2}(0,0,0), f_{3,3}(0,0,0))$$

$$(1,0,0) \mapsto (0,0,1,0) = (f_{3,0}(1,0,0), f_{3,1}(1,0,0), f_{3,2}(1,0,0), f_{3,3}(1,0,0))$$

$$(1,1,0) \mapsto (0,1,0,0) = (f_{3,0}(1,1,0), f_{3,1}(1,1,0), f_{3,2}(1,1,0), f_{3,3}(1,1,0))$$

$$(1,1,1) \mapsto (1,0,0,0) = (f_{3,0}(1,1,1), f_{3,1}(1,1,1), f_{3,2}(1,1,1), f_{3,3}(1,1,1))$$

$$H_3(x_0, x_1, x_2) = (f_{3,0}(x_0, x_1, x_2), f_{3,1}(x_0, x_1, x_2), f_{3,2}(x_0, x_1, x_2), f_{3,3}(x_0, x_1, x_2))$$

donde,

$$f_{3,0}(x_0, x_1, x_2) = x_2 = f_{2,0}(x_1, x_2)$$

$$f_{3,1}(x_0, x_1, x_2) = (1 - x_2)x_1 = (1 - f_{3,0}(x_0, x_1, x_2))f_{2,0}(x_0, x_1)$$

$$f_{3,2}(x_0, x_1, x_2) = (1 - x_2)(1 - x_1)x_0 = (1 - f_{3,0}(x_0, x_1, x_2))f_{2,1}(x_0, x_1)$$

$$f_{3,3}(x_0, x_1, x_2) = (1 - x_2)(1 - x_1)(1 - x_0) = (1 - f_{3,0}(x_0, x_1, x_2))f_{2,2}(x_0, x_1)$$

Para $k \in \mathbb{N} \cup \{0\}$, con $k \leq n$, sea la función $f_{n,k}: \{0,1\}^n \rightarrow \{0,1\}$ definida así:

$$f_{n,k}(x_0, \dots, x_{n-1}) = \begin{cases} x_{n-1}, & n = 1, k = 0 \\ 1 - f_{n,k-1}(x_0, \dots, x_{n-1}), & n = 1, k = 1 \\ f_{n,k}(x_0, \dots, x_{n-1}), & n > 1, k = 0 \\ (1 - f_{n,0}(x_0, \dots, x_{n-1}))f_{n-1,k-1}(x_0, \dots, x_{n-2}), & n > 1, k > 0 \end{cases}$$

Esto es,

$$f_{n,k}(x_0, \dots, x_{n-1}) = \begin{cases} x_{n-1}, & n \geq 1, k = 0 \\ 1 - x_{n-1}, & n = 1, k = 1 \\ (1 - x_{n-1})f_{n-1,k-1}(x_0, \dots, x_{n-2}), & n > 1, k > 0 \end{cases}$$

Luego, se define la función $H_n: F_n \rightarrow G_{n+1}$, así:

$$H_n(x_0, \dots, x_{n-1}) = (f_{n,0}(x_0, \dots, x_{n-1}), \dots, f_{n,n}(x_0, \dots, x_{n-1}))$$

Proyectores

Finalmente, se define la función $P_k^{n+1}: \mathbb{R}^{n+1} \rightarrow \mathbb{R}$, así:

$$P_k^{n+1}(y_0, \dots, y_n) = (y_0, \dots, y_n) \cdot H_n(x_0, \dots, x_{n-1}) = y_k$$

donde,

$$x_0 + x_1 + \dots + x_{n-1} = n - k$$